TM No. 861166

NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CONNECTICUT 06320

Technical Memorandum

THE ORIGIN OF DIFFERENT ACOUSTIC PERTURBATION SCATTERING CONCEPTS AT ROUGH RANDOM SURFACES

Date: 1 April 1985

Prepared by: Edward Y. T. Kuo
Edward Y. T. Kuo
Code 3322
Mechanical Engineering and
Acoustic Application Division
Surface Ship Sonar Department

REFERENCE ONLY

Report Documentation Page						Form Approved OMB No. 0704-0188
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.						
1. REPORT DATE **01 APR 1985**		2. REPORT TYPE **Technical Memo**			3. DATES COVERED **01-04-1985 to 01-04-1985**	
4. TITLE AND SUBTITLE **The Origin of Different Acoustic Perturbation Scattering Concepts at Rough Random Surfaces**					5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) **Edward Kuo**					5d. PROJECT NUMBER **733P82** 5e. TASK NUMBER 5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Underwater Systems Center,New London,CT,06320**					8. PERFORMING ORGANIZATION REPORT NUMBER **TM 861166**	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)					10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**						
13. SUPPLEMENTARY NOTES **NUWC2015**						
14. ABSTRACT **Recent translations from Russian literature show that the Russian community has analyzed acoustic wave scattering by utilizing Bass’s perturbation method. The method of Bass was initiated in 196O for the community of electromagnetic field about the time Marsh formulated his for the community of underwater acoustic field. The Russian community has since been aware of the concept of Marsh-Kuo. Somehow, Bass’s method got quite popular in the U.S. and England. It became the basis of some publications reporting solutions to problems basically addressed by the method of Marsh-Kuo ten years earlier. In this letter, an attempt is made to identify the common origin of these scattering concepts.**						
15. SUBJECT TERMS **scattering concepts; Bass method; Marsh-Kuo**						
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON	
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**24**		

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Cy001

PREFACE

This report was prepared under project 733P82. The author is grateful to valuable comments made by B. Cole, R. P. Radlinski, S. M. Rupinski, especially R. Deavenport at NUSC, New London.

1. Introduction

Since Marsh[1,2,3] proposed his acoustic scattering concept, the concept had been generally misunderstood. This author[4] had extended his theory to imperfect reflectors and found the theory very useful and physically revealing.

Recent translations[5,6] from Russian literature show that the Russian community has analyzed acoustic wave scattering by utilizing Bass's perturbation method.[7] The method of Bass was initiated in 1960 for the community of electromagnetic field about the time Marsh formulated his for the community of underwater acoustic field. The Russian community has since been aware of the concept of Marsh-Kuo.

Somehow, Bass's method got quite popular in the U.S. and England. It became the basis of some publications[8,9,10] reporting solutions to problems basically addressed by the method of Marsh-Kuo ten years earlier.

In this letter, an attempt is made to identify the common origin of these scattering concepts.

2. A Genetic Tree of Green

Let us start from the Green's identity (see e.g. Baker and Copson[11]) which takes the following form:

$$\iiint_V (\psi \nabla'^2 \phi - \phi \nabla'^2 \psi)\, d\underline{x}' = \iint_{S'} \left(\psi \frac{\partial \phi}{\partial N} - \phi \frac{\partial \psi}{\partial N}\right) dS' \qquad (1)$$

where S' is a closed surface bounding the volume V and $\partial/\partial N$

denotes differentiation along the outward normal to S'. This identity is valid when ϕ and ψ and their first- and second-order partial derivatives are continuous within and on S'. In particular if ϕ and ψ satisfy following Helmholtz equations:

$$(\nabla'^2 + k^2)\,\phi(\underline{x}') = -4\pi Q(\underline{x}') \tag{2}$$

$$(\nabla'^2 + k^2)\,\psi(\underline{x},\underline{x}') = -4\pi\,\delta(\underline{x}-\underline{x}') \tag{3}$$

Where $k = \omega/c$, $\underline{x} = (x, y, z)$, $\underline{x}' = (x', y', z')$

then, equation (1) is reduced to the Green's formula (e.g. Bass and Fuks[5]):

$$\phi(\underline{x}) = \iiint_V \psi(\underline{x}';\underline{x})\,Q(\underline{x}')\,d\underline{x}' + \frac{1}{4\pi}\iint_{S'}\left[\psi(\underline{x}';\underline{x})\frac{\partial\phi(\underline{x}')}{\partial N} - \phi(\underline{x}')\frac{\partial\psi(\underline{x}';\underline{x})}{\partial N}\right]dS' \tag{4}$$

In the first integral the region of integration is the Volume V, filled by sound sources. In the second integral the region of integration is the surface S', filled by a certain distribution of simple and double sources represented by ϕ and $\partial\phi/\partial N$ respectively.

When Q = 0, equation (4) becomes:

$$\phi(\underline{x}) = \frac{1}{4\pi}\iint_{S'}\left[\psi(\underline{x}';\underline{x})\frac{\partial\phi(\underline{x}')}{\partial N} - \phi(\underline{x}')\frac{\partial\psi(\underline{x}';\underline{x})}{\partial N}\right]dS' \tag{5}$$

in which $\underline{x}'$ is the location vector for the bounding surface points. The bounding surface S' can be divided into a local boundary surface, s, and a hemi-spherical surface at infinity (say S_∞). When Sommerfield conditions are satisfied (see e.g.

Baker and Copson[11]), equation (5) is reduced to the following Green's formula:

$$\phi(\underline{x}) = \frac{1}{4\pi}\iint_S \left[\psi(\underline{x}';\underline{x})\frac{\partial\phi(\underline{x}')}{\partial N} - \phi(\underline{x}')\frac{\partial\psi(\underline{x}';\underline{x})}{\partial N}\right] ds \tag{6}$$

To summarize, the genesis of equation (6) is the Green's identity. Equation (6) is a specialized form of the Green's identity for a wave field when sources are distributed only on a surface s. It simply says that the velocity potential field at any point in space is defined by the potential value ϕ and its normal derivative $\partial\phi/\partial N$ on the surface s. Equation (6), in turn, can be identified to be the genesis of existing surface scattering theories, while equation (4), without surface sources, is the genesis of existing volumetric scattering theories. Starting from Green's identity through Green's formula to wave scattering theories, it forms a kind of a genetic tree, say Green's Tree, of the origin of various scattering concepts.

3. Identification of Past Acoustic Scattering Concepts

Based on Green's formula (6), scattering concepts for rough boundaries can be identified with modern existing theories.

Let's identify the geometry of interest as in Figure 1. A general class of Green's function can be given by (e.g., Morse and Feshback[12]):

$$\psi(\underline{x},\underline{x}') = \frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|} + g_0(\underline{x},\underline{x}') \tag{7}$$

Where $|\underline{x} - \underline{x}'|^2 = (x - x')^2 + (y - y')^2 + (z - z')^2$.

According to properties of Green's function, the first term is the proper form of Green's function for a three dimensional

point source in free space. The second term accounts for the effect of boundaries.

Given an incident wave field in $z > z'$, a scattering theory seeks to estimate the effects of a rough boundary: $z' = \zeta(x', y')$. The incident wave field is assumed effectively plane, thus its effective source location is at ∞ . The Green's formula, equation (6), asserts that the reflected wave field can be obtained by summing the effective surface sources induced by the incident wave field on the rough surface s, according to the integral formula. In that integrand, ϕ represent the total field in the upper medium ($z > z'$) evaluated at the rough boundary, while Green's function $\psi(\underline{x}', \underline{x}) = \psi(\underline{x}, \underline{x}')$ is the solution to the differential equation (3) and is utilized to express the directivity pattern of these effective surface sources.

In a similar way, another Green's integral formula for the transmitted wave field in the lower medium ($z < z'$) can be derived. It will be of the same form as equation (6). However, ϕ in the integrand therein will be the transmitted wave field evaluated at the rough boundary and Green's function will satisfy a differential equation like equation (3) with appropriate k for the lower medium.

Basically then, the above discussion implies that the rough boundary can be replaced by a distribution of surface sources of the appropriate strength and directivity. In this respect, these effective surface sources are in free space and the form of Green's function without g_o in equation (7) is quite general for the study of acoustic wave scattering at rough boundaries.

By assuming Green's function in the spectral form:

$$\psi(\underline{x}, \underline{x}') = \iiint e^{i\underline{K}\cdot\underline{x}} \; A(\underline{K}, \underline{x}')\, d\underline{K} \tag{8}$$

Where $\underline{K} = (k_x, k_y, k_z)$

A ($\underline{K}$, $\underline{x}$') is found from equation (3) to be:

$$A(\underline{K},\underline{x}') = -\frac{2}{(2\pi)^2}\,\frac{e^{-i\underline{K}\cdot\underline{x}'}}{k^2-|\underline{K}|^2} \tag{9}$$

i.e.

$$\psi(\underline{x},\underline{x}') = -\frac{2}{(2\pi)^2}\iiint e^{i\underline{K}\cdot(\underline{x}-\underline{x}')}\,\frac{d\underline{K}}{k^2-|\underline{K}|^2} \tag{10}$$

It can be shown that integration of the above equation produces the result:

$$\psi(\underline{x},\underline{x}') = \frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|} \tag{11}$$

This Green's function in three dimensional free space can also be obtained from the physical nature of Green's function (see e.g. Morse and Feshback[12]). The method selected here is to illustrate the tie between modern scattering theory and Green's formula.

It can also be shown that integration with respect to k_z only yields the result:

$$\psi(\underline{r}, \underline{r}') = \frac{i}{2\pi} \iint e^{i[k_x(x-x')+k_y(y-y')\pm k_z(z-z')]} \frac{dk_x dk_y}{k_z} \tag{12}$$

for $z - z' > o$ and $z - z' < o$, respectively. That is

$$\frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} = \frac{i}{2\pi} \iint e^{i[k_x(x-x')+k_y(y-y')\pm k_z(z-z')]} \frac{dk_x dk_y}{k_z} \tag{13}$$

for $z - z' > 0$ and $z - z' < 0$, respectively. A different method was utilized by Brekhovskikh[13] to derive a similar equation for which $z' = o$.

Noting that the inward normal $\hat{n}$ $(= - \hat{N})$ in Figure 1, is given by

$$\hat{n} = (-\zeta_{x'}, -\zeta_{y'}, 1) / [1 + \zeta_{x'}^2 + \zeta_{y'}^2]^{1/2} \tag{14}$$

the following expression can be obtained from equation (13):

$$\frac{\partial}{\partial n'} \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} = \frac{i}{2\pi} \iint e^{i[k_x(x-x')+k_y(y-y')+k_z(z-z')]} \cdot$$

$$\cdot \frac{i}{[1+\zeta_{x'}^2+\zeta_{y'}^2]^{1/2}} [k_x \zeta_{x'} + k_y \zeta_{y'} - k_z] \frac{dk_x dk_y}{k_z} \tag{15}$$

for $z - z' > 0$, i.e. the upper medium.

The following relationship transforms ds to

$dA = dx' \, dy'$:

$$ds = [1+\zeta_{x'}^2+\zeta_{y'}^2]^{1/2}\, dx'dy' \tag{16}$$

Utilizing equation (11), the Green's formula (6) can be written as:

$$\phi(\underline{x}) = -\frac{1}{4\pi}\iint_S \left[\frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|}\frac{\partial\phi(\underline{x}')}{\partial n'} - \phi(\underline{x}')\frac{\partial}{\partial n'}\frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|}\right] ds \tag{17}$$

This is the well known theorem of Helmholtz or the Helmholtz integral (e.g., Baker and Copson[11]). This integral form of solution has been a popular starting point of published studies in high frequency scattering by using Kirchhoff's local plane approximation (see e.g., Eckart[14], and Mintzer[15]).

Utilizing equation (13), (15), and (16), equation (17) takes the following form:

$$\phi(\underline{x}) = \iint_{k_x,k_y} e^{i[k_x x + k_y y + k_z z]}\, \frac{1}{(2\pi)^2}\,\cdot$$

$$\cdot\iint_{x',y'} e^{-i[k_x x' + k_y y']}\, F(x',y',k_x,k_y)\, dx'dy'dk_x\, dk_y \tag{18}$$

Where

$$F(x', y'; k_x, k_y) = \frac{i}{2k_z} e^{-ik_z \zeta(x', y')} \cdot \Big[\zeta_{x'} \frac{\partial \phi(z')}{\partial x'} \tag{19}$$

$$+ \zeta_{y'} \frac{\partial \phi(z')}{\partial y'} - \frac{\partial \phi(z')}{\partial z'} + \phi(z') i (k_x \zeta_{x'} + k_y \zeta_{y'} - k_z) \Big]_{z' = \zeta(x', y')}$$

By defining the following Fourier Integral Transform

$$\tilde{F}(k_x, k_y) = \frac{1}{(2\pi)^2} \iint_{x', y'} e^{-i(k_x x' + k_y y')} F(x', y'; k_x, k_y)\, dx'\, dy' \tag{20}$$

equation (18) can be written as

$$\phi(x,y,z) = \iint e^{i[k_x x + k_y y + k_z z]} \tilde{F}(k_x, k_y)\, dk_x\, dk_y \tag{21}$$

It is to be noted that $\tilde{F}(k_x, k_y)$ is a Fourier Integral Transform of a complicated statistical function of the roughness characteristics represented by quantities like ζ, $\zeta_{x'}$, and of the boundary properties indirectly represented by quantities like $\phi(\underline{x}')$, $\partial\phi(\underline{x}')/\partial x'$, that are affected by the densities and acoustic velocities of the contacting media.

According to equation (13) representing a quasi-plane wave expansion, equation (21) implies the following. The total contribution from an equivalent plane source distribution given by F, equation (19), to the reflected wavefield in the upper medium, ($z > z'$) at an observation point $\underline{x}$, can be expressed as the combined effects from quasi-plane waves. Amplitudes of these quasi-plane waves, $\tilde{F}$, are a function of radiation directions (k_x, k_y), boundary characteristics, and the incident wave field. They are quasi-plane wave because many radiation directions are of the imaginary decaying type.

A similar process will lead to an integral form similar to equation (21) for the transmitted wave field in the lower medium ($z < z'$).

These are exactly the plane wave expansion forms utilized as a starting point for the development of surface scattering theory

by Marsh et al [1,2,3] and Kuo[4]. Thus, equation (21) is the Quasi Fourier equivalent of the Helmholtz integral.

In the form of equation (21), the effects of incident wave and boundary characteristics (both roughness and acoustic properties) on the reflected field are seen to be expressed in terms of their total effects on each Generalized Fourier Component of the reflected field. Each Generalized Fourier component represents a reflected field in a different reflected direction. At this point, the perturbation concept of Marsh-Kuo proposed the expansion of this random Fourier component amplitude in terms of the pertinent statistical properties consistent with boundary conditions of interest.

Another basis to modern surface scattering theories came from Bass[7,5] and can be divided into two types.

The original scattering concept advanced by Bass [7] was to express the reflected field as the sum of mean and stochastic fields. Then he applied small perturbation to and performed statistical average of the appropriate boundary conditions. The latter process produced two boundary equations or conditions. The first equation was the mean field boundary equation in which the mean field at the boundary was related to mean statistical properties of the boundary and to correlation between random roughness and the stochastic field at the boundary. The second equation was the stochastic field boundary equation in which the stochastic field. at the boundary was related to random boundary roughness variables and the mean field at the boundary. This - stochastic field at the boundary was identified as the surface boundary source to the stochastic field in space through the use of the Green's formula (6) with an appropriate Green's function evaluated at the mean surface. The resulting Green's integral equation, then, expressed the stocahstic field in space in terms of a mean field at the boundary and random boundary characteristics. By utilizing this stochastic integral equation,

the stochastic field in the mean field boundary equation (the first boundary equation) can be eliminated. Thus, the mean field at mean boundary is an integral representation like the Helmholtz integral whose surface source is a function of the mean field and mean roughness properties at the mean boundary. Based on this Helmholtz integral representation of the mean field and identifying the mean field as that of the well known field produced by the plane boundary, quantities such as reflectivity can be computed.

The above type of concept formed the basis of problems of perfect reflectors reported by Wenzel[8] and by Dowling et al[10].

Later in his book[5], Bass reported a change in his method (possibly reported originally by Alekhin[16]). The only change was to use the plane wave expansion form as (21) to continue his stochastic field at the boundary into space. Thus the new method incorporated the Quasi-Fourier type approach, or the plane wave expansion approach. Analyses based on this type of approach were published by Lysanov[17] and Kuperman[9].

The modified Bass method or even the old method, is not really different from that of Marsh-Kuo in perturbation principle. The latter method starts from expressions like (21) for reflected and transmitted fields. Then $\tilde{F}(k_x, k_y)$ for each fields, was found by formally satisfying the required boundary conditions to the second order in the boundary roughness characteristics. $\tilde{F}(k_x, k_y)$ for each field thus found is substituted back into the form of solutions like (21). The results are the total reflected and trnsmitted fields. A statistical average of these results may be identified with the averaged field solution of Bass et al.

So far, the assumed form of solution (21) utilized by Marsh-Kuo, was shown to be the Quasi-Fourier equivalent of Helmholtz integral and as such an exact solution. It has been

also briefly indicated how the Bass scattering theories are genetically located in Green's tree relative to Marsh-Kuo's theory. In principle, one should be able to show how they are related. However, because of different approaches, fully spectral approach of Marsh-Kuo on one hand and fully spatial or mixed spatial-spectral approaches on the other hand, assessment of the equivalence is difficult without detail analysis. This is not in the scope of this letter. However, we shall show in the following the equivalence to the first order in the boundary characteristics.

Dowling et al[10] considered an incident wave of time dependence $\exp(-i\omega t)$. It was assumed to propagate in the general direction against vertical coordinate z and in the same general direction of horizontal coordinates (x,y). The scattering phenomenum of this wave from an underlining rigid and rough boundary located at z=0, was investigated.

By following Bass, the total potential ϕ was divided into a mean field $\bar{\Phi}$ and a stochastic field ϕ', i.e., $\phi = \bar{\Phi} + \phi'$. The rigid rough boundary condition was perturbed and statistically averaged. The result was then subtracted from the rigid boundary condition. The result is the following.

$$\frac{\partial \phi'}{\partial z'} = -h\zeta(x',y')\frac{\partial^2 \bar{\Phi}}{\partial z'^2} + s\zeta_{x'}(x',y')\frac{\partial \bar{\Phi}}{\partial x'} + s\zeta_{y'}(x',y')\frac{\partial \bar{\Phi}}{\partial y'} \tag{22}$$

at $z'=0$, where

$h\zeta =$ random roughness $\qquad$ $s\zeta_{x',y'} =$ random slopes

To continue the stochastic boundary condition into space from the boundary, Green's formula in the form of the following equation was derived.

$$\phi'(x,y,z) = \iint_A \left(\frac{-1}{2\pi}\right) \frac{e^{ik|\underline{x}-\underline{x}'|}}{|\underline{x}-\underline{x}'|} \frac{\partial \phi'}{\partial z'} dx'dy' \tag{23}$$

Kuo[4] represented the same field as the sum of incident field ϕ_{in} and the reflected field ϕ_r, i.e.,

$$\phi = \phi_{in} + \phi_r \tag{24}$$

$$\phi_{in} = \exp\left[i\left(\underline{k}_1 \cdot \underline{x} - k\gamma z\right)\right] \tag{25}$$

$$\phi_r = \iint \exp\left[i\left(\underline{k}\cdot\underline{x} + k\nu z\right)\right] \cdot \left[dA_{00}(\lambda,\mu) + h\,dA_{10}(\lambda,\mu) + s\,dA_{01}(\lambda,\mu) + h^2 dA_{20}(\lambda,\mu) + \cdots\right] \tag{26}$$

Where

$$\underline{k}_1 = (k\alpha, k\beta), \quad \underline{k} = (k\lambda, k\mu), \quad \underline{x} = (x, y).$$

To the first order in boundary characteristics, $\bar{\phi}$ will be shown later to be the sum of the incident field potential, ϕ_{in}, and the specularly reflected field potential. Therefore, $\bar{\phi}$ is the total potential field above a flat boundary located at the mean rough boundary. This identification is only true to the first order. To the second order, the specularly reflected component contains roughness dependent terms which estimate the energy loss from specular direction to all scattering directions.

The stochastic field potential, ϕ', will be identified as that of randomly scattered field generated by the boundary roughness and needs statistical analysis for its physical implications.

The form of ϕ_r is the expanded form of equation (21) in terms of roughness height, h, and slope, s. This particular way of expansion is required by the rigid boundary condition. Substituting ϕ_{in} and ϕ_r in the rough boundary condition, followed by perturbation in term of the roughness characteristics, the following results were obtained:

$$dA_{00}(\lambda,\mu) = \delta(\lambda-\alpha,\mu-\beta)\,d\lambda\,d\mu \tag{27}$$

$$h\,dA_{10}(\lambda,\mu) = \frac{-i}{(2\pi)^2}\iint_{\underline{x}'} \exp\left[-i(\underline{k}-\underline{k}_1)\cdot\underline{x}'\right]\cdot$$

$$\cdot\, h\zeta(\underline{x}')\,d\underline{x}'\,2k^2\gamma^2\,d\underline{k}/k_z \tag{28}$$

$$s\,dA_{01}(\lambda,\mu) = \frac{-i}{(2\pi)^2}\iint_{\underline{x}'} \exp\left[-i(\underline{k}-\underline{k}_1)\cdot\underline{x}'\right]\cdot$$

$$\cdot\left[i\alpha s\zeta_{x'}(\underline{x}')+i\beta s\zeta_{y'}(\underline{x}')\right]d\underline{x}'\,2k\,d\underline{k}/k_z \tag{29}$$

Therefore ϕ_r is given by

$$\phi_r(\underline{x},z) = \exp\left[i(\underline{k}_1\cdot\underline{x}+k\gamma z)\right]$$

$$+\iint \exp\left[i(\underline{k}\cdot\underline{x}+k\nu z)\right]\frac{-i}{(2\pi)^2}\iint_{\underline{x}'}\exp\left[-i(\underline{k}-\underline{k}_1)\cdot\underline{x}'\right]\cdot$$

$$\cdot 2\left[k^2\gamma^2 h\zeta(\underline{x}') + ik\alpha s\zeta_{x'}(\underline{x}') + ik\beta s\zeta_{y'}(\underline{x}')\right] d\underline{x}'\, d\underline{k}/k_z \tag{30}$$

to the first order in h and s. The previous results were modified to reflect the changes in the incident field geometry, time dependence, and the change in the boundary conditin to rigid boundary.

Accordingly, from equations (24), (25), and (30) the following expressions can be obtained.

$$\bar{\Phi} = \exp[i(\underline{k}_i\cdot\underline{x} - k\gamma z)] + \exp[i(\underline{k}_i\cdot\underline{x} + k\gamma z)] \tag{31}$$

$$\phi' = \phi - \bar{\Phi} = \phi_{in} + \phi_r - \bar{\Phi}$$

$$= \iint \exp[i(\underline{k}\cdot\underline{x} + k\nu z)] \frac{-i}{(2\pi)^2} \iint_{\underline{x}'} \exp[-i(\underline{k}-\underline{k}_i)\cdot\underline{x}'] \cdot$$

$$\cdot 2\left[k^2\gamma^2 h\zeta(\underline{x}') + ik\alpha s\zeta_{x'}(\underline{x}')\right.$$

$$\left. + ik\beta s\zeta_{y'}(\underline{x}')\right] d\underline{x}'\, d\underline{k}/k_z .$$

(32)

Substitution of equation (31) into (22) results in the expression

$$\left.\frac{\partial \phi'}{\partial z'}\right|_{z'=0} = 2\left[k^2 \gamma^2 h \zeta(\underline{x}') + i k \alpha s \zeta_{x'}(\underline{x}') + i k \beta s \zeta_{y'}(\underline{x}')\right] \exp\left[i \underline{k}_1 \cdot \underline{x}'\right] \tag{33}$$

By utilizing equation (13), equation (23) can be recast in the form:

$$\phi'(x,y,z) = -\frac{i}{(2\pi)^2} \iint \exp\left[i(\underline{k}\cdot\underline{x} + k\nu z)\right] \cdot \iint_{\underline{x}'} \exp\left[-i\underline{k}\cdot\underline{x}'\right] \left.\frac{\partial \phi'}{\partial z'}\right|_{z'=0} d\underline{x}'\, d\underline{k} / k_z \tag{34}$$

Substitution of relationship (33) in the above equation will yield the same result as in equation (32).

This proves the equivalence of two perturbation methods to the first order in the random roughness. However, the total equivalence requires the analysis to at least second order in the statistical properties, e.g., random roughness and random slopes.

4. Conclusions

Although two perturbation scattering theories originally formulated by Marsh[1] and Bass[7] appear to have the same mathematical approximation, there is a basic difference.

In the Bass's method, the potential field is perturbed at the start by utilizing power or Taylor series. The approximate solution is then substituted into the approximate boundary conditions perturbed with respect to the random roughness. The perturbed field at the boundary that satisfies the approximate boundary conditions is continued into space by either using Green's formula (or Helmholtz integral) or using its Quasi-Fourier equivalent form of the quasi-plane wave expansion.

In the Marsh's method, the exact solution in the form of quasi-plane wave expansion is substituted into the appropriate exact boundary conditions. The scattered field at the boundary is then solved by utilizing perturbations in terms of the boundary characteristics now apparent from the boundary conditions.

The Marsh's method is preferred because perturbations are not

required at the beginning when the required types of perturbations are not apparent.

References

1. H. W. Marsh, "Exact Solution of Wave Scattering by Irregular Surfaces", J. Acoust. Soc. Am., 33, 330-333 (1961).

2. H. W. Marsh, M. Schulkin, and S. G. Kneale, "Scattering of Underwater Sound by the Sea Surface", J. Acoust. Soc. Am., 33, 334-340 (1961).

3. H. W. Marsh, "Sound Reflection and Scattering from the Sea Surface", J. Acoust. Soc. Am., 35, 240-244 (1963).

4. E. Y. T. Kuo, "Wave Scattering and Transmission at Irregular Surfaces", J. Acoust. Soc. Am., 36, 2135-2142 (1964).

5. F. G. Bass and I. M. Fuks, "Wave Scattering from Statistically Rough Surfaces", Wauka Press, Moscow, 1972 (in Russian), Translated by C. B. Vesecky and J. F. Vesecky, Pergamon Press, 1979 (in English).

6. L. M. Brekhovskikh, "Acoustics of the Ocean", Wauka Press, Moscow, 1974 (in Russian), Translated by Department of the Navy, 1978.

7. F. G. Bass, "Boundary Conditions for the Average Electromagnetic Field on a Surface With Random Irregularities and With Impedance Fluctuations", Izv. Vuzov, Radio Fizika, 3, 1, 72-78 (1960)

8. A. R. Wenzel, "Smoothed Boundary Conditions for Randomly Rough Surfaces", J. Math. Phys., 15, 3, 317-323 (1974).

9. W. A. Kuperman, "Coherent Component of Specular Reflection and Transmission at a Randomly Rough Two-Fluid Interface", J. Acoust. Soc. Am., 58, 2, 365-370 (1975)

10. A. P. Dowling, D. G. Crighton, J. E. Ffowcs-Williams, and F. G. Leppington, "Aero and Hydro Acoustics: An Advanced Short Course", 1985.

11. B. B. Baker and E. T. Copson, "The Mathematical Theory of Huygen's Principle", Clarendon Press, Oxford, England, 1953.

12. P. M. Morse and H. Feshback, "Methods of Theoretical Physics", McGraw-Hill, (1953)

13. L. M. Brekhovskikh, "Waves in Layered Media", Translated by D. Lieberman, Academic Press, 1960.

14. C. Eckart, "The Scattering of Sound from the Sea Surface", J. Acoust. Soc. Am., 25, 566-570 (1953).

15. D. Mintzer, "Discussion of the Paper by C. Eckart on Sea Surface Scattering", J. Acoust. Soc. Am., 25, 1015 (1953).

16. Y. K. Alekhin, "The Formulation of a Surface Wave Near a Rough Interface Between Two Media", Soviet Physics-Dokl., 14, #11, 1095-1097 (1970)

17. Y. P. Lysanov, "Mean Coefficient of Reflection from an Uneven Surface Bounding an Imhomogeneous Medium", Soviet Physics-Acoustics, 15, 3, 340-344 (1970).

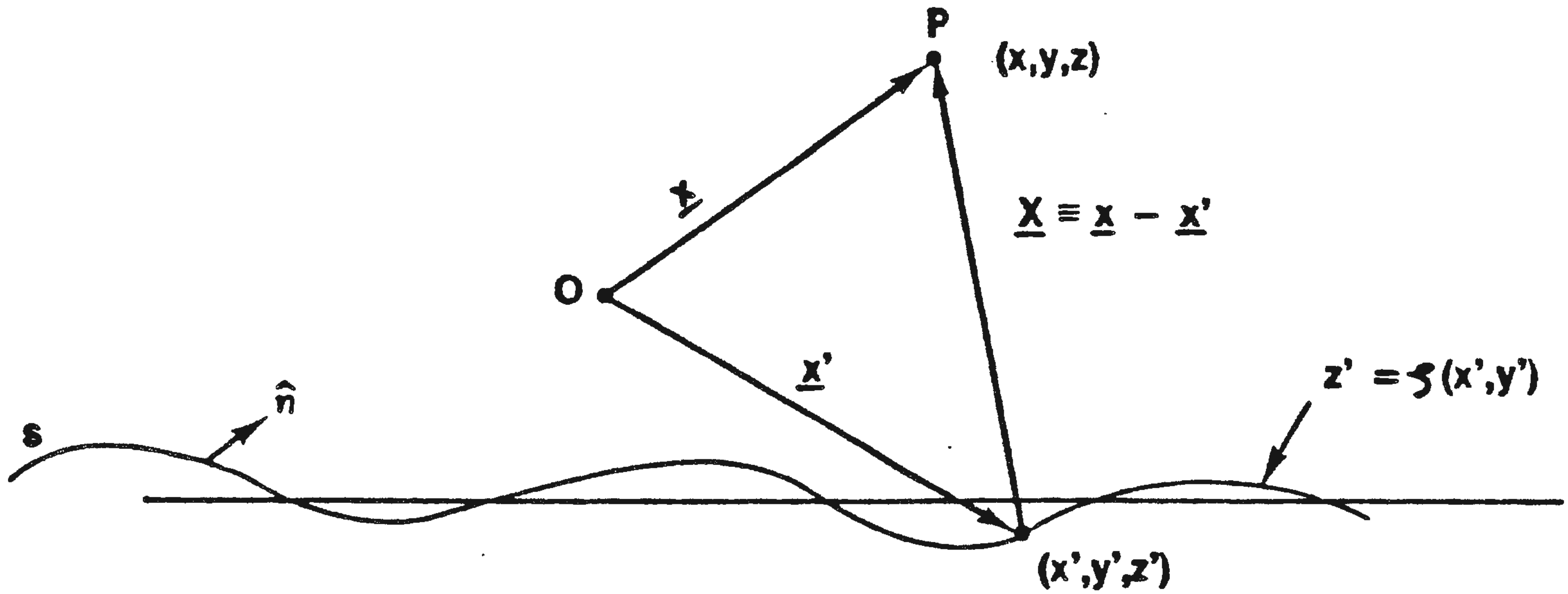

FIGURE 1 SCATTERING BOUNDARY

THE ORIGIN OF DIFFERENT ACOUSTIC PERTURBATION
SCATTERING CONCEPTS AT ROUGH RANDOM SURFACES
E. Y. T. Kuo
Mechanical Engineering and
Acoustic Application Division
TM. No. 861166
1 April 1985
Distribution Statement A
UNCLASSIFIED

DISTRIBUTION LIST

<u>Internal</u>
Codes: 021311 (Library)
01X (R. Chapman)
01Y (D. McKinley, J. Donald)
10 (W. VonWinkle, K. Lima, R. Kasper)
21 (J. Kingsbury, W. Roderick)
33 (L. Freeman, B. Cole, P. Herstein, G. Mayer, W. Carey)
332 (R. Radlinski)
3322 (S. Rupinski, E. Kuo (3))
3331 (M. Tattersall)
3332 (R. Deavenport, E. Jensen)

<u>Total</u>: 22